

IMPACTFUL LEARNING DESIGNS

Learning Progression for Basic Addition

As young children develop conceptual understanding and move towards mastering basic addition and its related properties, they move through a three-phase learning progression, plus the related prerequisite skills. The graphic below highlights this learning progression and can inform teachers on the types of learning experiences they should design to accelerate student learning.

The learning progress was informed by the following sources:

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics*. Pearson.

Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, 13(1), 22-31.

Henry, V. J., & Brown, R. S. (2008). First-grade basic facts: An investigation into teaching and learning of an accelerated, high-demand memorization standard. *Journal for Research in Mathematics Education*, 39(2), 153-183.

Prerequisite Skills

Counting Sequence: Able to produce the list of counting words in order “One, two, three...”

One-to-one Correspondence: being able to touch and count each object only once in a set

Phase 1—Counting Strategies

Direct Modeling

Counting objects or fingers

Counting All

Counting the two sets of objects and then joining the objects into one combined set

Counting On from First

Knows the number of the first set of objects and then counts on for the other set.

Counting On from Larger

Starts with larger number in the two sets and then counts on the smaller set of objects

Counting Abstractly

Counting without objects

Counting All

Counting the two sets of objects and then joining the objects into one combined set

Counting On from First

Knows the number of the first set of objects and then counts on for the other set.

Counting On from Larger

Starts with larger number in the two sets and then counts on the smaller set of objects

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Phase 2—Reasoning Strategies

KNOWLEDGE OF PROPERTIES

Zero Property

Knows when zero is being added that the sum will not **change** (e.g., $a + 0 = a$)

Commutative Property

Knows the order of numbers being added will not change the sum (e.g., $3 + 5 = 5 + 3$)

Associative Property

Knows that when adding three or more addends that it doesn't matter which pair is added first

EXPLICIT STRATEGIES

Adding with one or two

Knows when adding one or two it is just the next number or two in the counting sequence

5 as an Anchor

Sees a number under 10 as $5 + \underline{\quad}$
(e.g., thinks of 7 as $5 + 2$)

Making 10

Knows the combinations that equal 10
(e.g., 4 and 6, 3 and 7)

Making 10 and Adding On

Uses their combinations for making 10 to solve sums greater than 10

Known Fact Derivations

Uses already known facts to derive new facts
(e.g., knows $3 + 3 = 6$, so $3 + 4 = 7$)

Redistributed Derived Facts

Breaks down the addends into known facts to solve an unknown fact (e.g., for $7 + 5$, the student might use the known fact $5 + 2 = 7$ and then think $5 + 5 + 2$)

Phase 3—Mastery

Produces answers fluently. Fluently means:

- Accurate: answer is correct
- Efficient: time needed to solve is reasonable
- Strategy selection: strategy chosen in appropriate
- Flexibility: applies strategies to new problems or adapts to better fit the problem